

An Alternate to the Unified Distribution with Application to Breast Cancer Data

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ABSTRACT

The Unified (θ) distribution appeared in [1], and in this short note we introduce a variant of this distribution. Further, a member of this family is shown to be significant in cancer modeling.

KEYWORDS: Bates distribution; Unified distribution; Cancer modeling

THE UNIFIED (θ) DISTRIBUTION

For $x \in (0,1)$, the PDF is given by

$$f\{x; \theta\} = \begin{cases} \frac{\theta e^{x\theta}}{e^\theta - 1} & \text{if } \theta \neq 0 \\ 1 & \text{if } \theta = 0 \end{cases}$$

Remark 3.1. For details on the derivation of this family, see [1]

THE NEW FAMILY

Let X be Bates on $(0, 1)$ [2], and denote its PDF as $f_x(x, n)$ and consider

$$q(\phi, x, \theta) = \left(x\phi, \frac{1}{\phi}\right) \exp\{x\phi - k(\theta)\}$$

where k and ϕ are defined as in [1]. Suppose $Q=q$, and consider $=a+(b-a)J$, where the random variable J has density q then

$$P(Y \leq y) = P\left(J \leq \frac{y-a}{b-a}\right)$$

So, the CDF of Y is

$$Q\left(\phi, \frac{y-a}{b-a}, \theta\right)$$

By differentiation the PDF of Y is

$$\frac{1}{b-a} q\left(\phi, \frac{y-a}{b-a}, \theta\right)$$

So, if $\phi=1$, then our density becomes

$$v(y; a, b, \theta) = \begin{cases} \theta e^{\left(\frac{y-a}{b-a}\right)\theta} & \text{if } \theta \neq 0; y \in [a, b] \\ \frac{1}{b-a} & \text{if } \theta = 0; y \in [a, b] \end{cases}$$

The class of all distributions with the above density is denoted *A Unified* (a, b, θ)

Remark 4.1. The families *A Unified* (a, b, θ) and Unified (θ) (of Section 1) coincide if and only if $a=0$ and $b=1$

THE UNIFIED (θ) GENERATED FAMILY OF DISTRIBUTIONS

The *A Unified*(a, b, θ) generated family of distributions has CDF

$$M(x; \theta) = \int_0^{K(x)} \frac{\theta e^{t\theta}}{e^\theta - 1} dt, x \in \text{Supp}(K(x))$$

where $0 \neq \theta \in \mathbb{R}$, and $K(x)$ is the CDF of some baseline distribution. Assuming the baseline distribution is Weibull with the following CDF

$$K(x; g, h) = 1 - e^{-\left(\frac{x}{h}\right)^g}$$

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for $x, g, h > 0$, then from the integral we have the following

Proposition 5.1. The CDF of the A Unified $(0, 1, \theta)$ Weibull family is given by

$$M(x; \theta, g, h) = \frac{e^{\theta} \left(1 - e^{-\left(\frac{x}{h}\right)^g} \right) - 1}{e^{\theta} - 1}$$

where $x, g, h > 0$ and $0 \neq \theta \in \mathbb{R}$

Obviously, the PDF can be obtained by differentiation. We write $H \sim AUW(\theta, g, h)$, if H is a *A Unified* (a, b, θ) Weibull random variable. The family in the above Proposition is a good fit to real life data as shown below. The MLE estimates are obtained using the software MATHEMATICA (Figure 1) [3].

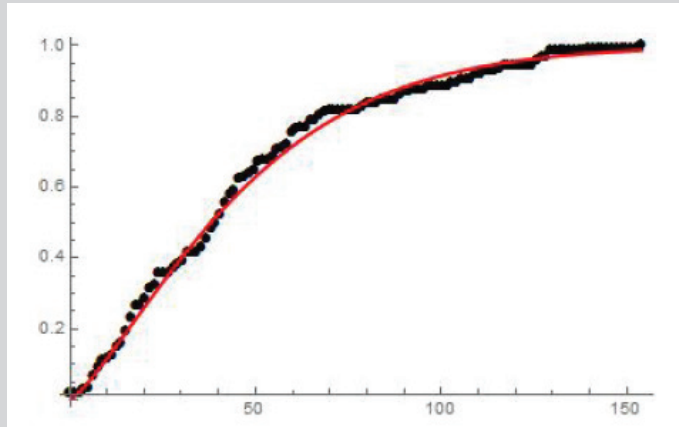


Figure 1: The CDF of AUW $(0.145698, 1.2849, 48.5111)$ fitted to the empirical distribution of the breast cancer data [3].

CONCLUDING REMARKS

In the present paper we introduced an alternate to the Unified distribution appearing in [1], and showed the Unified distribution is good in fitting real life data.

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